‘Where there is matter,
there is geometry.’

–**Johannes Kepler**

MATHEMATICS

LESSON PLAN

GRADE 12 TERM 4

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE). We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

WHAT IS NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

WHAT ARE THE LEARNING PROGRAMMES?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Universalisation Programme and in its Provincialisation Programme.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

CONTENTS

PROGRAMME ORIENTATION

Welcome!

The NECT FET Mathematics Learning Programme is designed to support teachers by providing:

- Lesson Plans
- Trackers
- Resource Packs
- Assessments and Memoranda
- Posters.

This Mathematics Learning Programme provides most of the planning required to teach FET Mathematics. However, it is important to remember that although the planning has been done for you, preparation is key to successful teaching. Set aside adequate time to properly prepare to teach each topic.

Also remember that the most important part of preparation is ensuring that you develop your own deep conceptual understanding of the topic. Do this by:

- working through the lesson plans for the topic
- watching the recommended video clips at the end of the topic
- completing all the worked examples in the lesson plans
- completing all activities and exercises in the textbook.

If, after this, a concept is still not clear to you, read through the section in the textbook or related teacher's guide, or ask a colleague for assistance. You may also wish to search for additional teaching videos and materials online.

Orientate yourself to this Learning Programme by looking at each component, and by taking note of the points that follow.

TERM 4 TEACHING PROGRAMME

1. In line with CAPS, the following teaching programme has been planned for FET Mathematics for Term 3:

Grade 10		Grade 11		Grade 12	
Topic	**No. of weeks**	**Topic**	**No. of weeks**	**Topic**	**No. of weeks**
Probability	2	Statistics	3	Revision	3
Revision	4	Revision	3		

2. Term 4 lesson plans and revision plan are provided for six weeks for Grades 10 and 11.
3. Term 4 Revision Plan are provided for three weeks for Grade 12
4. Each week includes 4,5 hours of teaching time, as per CAPS.
5. You may need to adjust the lesson breakdown to fit in with your school's timetable.

THE REVISION PROGRAMME

The teaching programme for FET mathematics Term 4 differs from the teaching programmes for Terms 1-3. There is only one topic with new content in Term 4 for Grades 10 and 11; and no new content in Term 4 for Grade 12. Most of the contact time in Term 4 is allocated to consolidation, revision and preparation for the end of year examinations. The Revision Programme for each grade are designed to support you and the learners so as to ensure that revision time is effectively and productively used.

THE STRUCTURE OF THE REVISION PROGRAMME

- Summary notes for the topics assessed in Paper I and Paper II. The summary notes are provided in the Resource Pack. If possible, the summary notes should be photocopied for learners. Alternatively, you could provide learners with an electronic copy of the summary notes; or learners can copy down the summary notes. Encourage learners to add their own notes to the summary notes you have given them.
- Fully worked past paper
- Past papers and memoranda. The past papers and memoranda are provided in the Resource Pack. If possible, the past papers, exemplars and memoranda should be photocopied for learners. Alternatively, you could provide learners with an electronic copy of the examinations, exemplars and memoranda; or learners can share copies. The links to these resources are provided in the Lesson Plan.

Working through past papers and exemplars has been shown to be an excellent learner-centred approach to revision. For this reason, we urge you to do everything possible to ensure that learners have access to these materials.

RESOURCE PACK, ASSESSMENT AND POSTERS

1. A Resource Pack with printable resources has been provided for each term.
2. These resources are referenced in the lesson plans, in the Classroom Management section.
3. Two posters have been provided as part of the FET Mathematics Learning Programme for Term 4.
4. Ensure that the posters are displayed in the classroom.
5. Try to ensure that the posters are durable and long-lasting by laminating it, or by covering it in contract adhesive.
6. Note that you will only be given these resources once. It is important for you to manage and store these resources properly. You can do this by
   - Writing your school's name on all resources
   - Sticking resource pages onto cardboard or paper
   - Laminating all resources, or covering them in contact paper
   - Filing the resource papers in plastic sleeves once you have completed a topic.
7. Add other resources to your resource file as you go along.
8. Note that these resources remain the property of the school to which they were issued.

ASSESSMENT AND MEMORANDUM

In the Resource Pack you are provided with assessment exemplars and memoranda as per CAPS requirements for the term. For Term 4, the Resource Pack contains one test and memorandum for Grades 10 and 11. In addition, past papers, exemplars and memoranda are provided for Grades 10, 11 and 12.

CONCLUSION

Teacher support and development is a complex process. For successful Mathematics teachers, certain aspects of this Learning Programme may strengthen your teaching approach. For emerging Mathematics teachers, we hope that this Learning Programme offers you meaningful support as you develop improved structure and routine in your classroom, develop deeper conceptual understanding in your learners and increase curriculum coverage.

Term 4

REVISION OVERVIEW

A. TOPIC OVERVIEW

A

- The Revision Plan is not presented over specific lessons. Guidance is given of what to complete in each week. Plan according to your own learners needs.
- Learners will write two examinations in November. Each examination is three hours long and 150 marks. Encourage learners to be well prepared.
- Use these revision weeks well. Learners need to feel confident when writing their final examinations.
- The revision programme is made up of three parts:
  - Summary notes to share with learners
  - One full Paper 1 and Paper 2 (2017) to work through with learners in detail
  - One Paper 1 and Paper 2 (exemplars) for learners to work on in class and at home and make 'cheat sheets' (their own summaries) at the same time.

Breakdown of revision programme:

Week 1	Paper 1 summary notes and past Paper 1
Week 2	Paper 2 summary notes and past Paper 2
Week 3	Paper 1 and Paper 2 + 'cheat sheets'.

As part of the revision programme, learners will work through past papers. This has been shown to be an excellent learner-centred approach to revision. In addition to providing the past papers and memoranda in the Resource Pack, we also provide the following links:

Links for past papers and memoranda:	
Paper 1 2017	https://www.education.gov.za/Portals/0/CD/Computer/2017%20November%20past%20papers/Non%20Languages/Mathematics%20P1%20Nov%202017%20Eng.pdf?ver=2018-03-07-103536-000
Paper 1 2017 memo	https://www.education.gov.za/LinkClick.aspx?fileticket=BvPSes02Sn4%3d&tabid=1856&portalid=0&mid=7350
Paper 2 2017	https://www.education.gov.za/Portals/0/CD/Computer/2017%20November%20past%20papers/Non%20Languages/Mathematics%20P2%20Nov%202017%20Eng.pdf?ver=2018-03-07-103150-000

Paper 2 2017 memo	https://www.education.gov.za/LinkClick.aspx?fileticket=zp4zBOSLW0Q%3d&tabid=1856&portalid=0&mid=7350
Paper 1 EXEMPLAR	http://maths.stithian.com/Gr12%20Exemplars/Gov%20Exemplars/DOE%20examplar%20P1%202014.pdf
Paper 1 MEMO EXEMPLAR	http://maths.stithian.com/Gr12%20Exemplars/Gov%20Exemplars/DOE%20examplar%20P1%202014%20MEMO.pdf
Paper 2 EXEMPLAR	http://maths.stithian.com/Gr12%20Exemplars/Gov%20Exemplars/DOE%20examplar%20P2%202014.pdf
Paper 2 MEMO EXEMPLAR	http://maths.stithian.com/Gr12%20Exemplars/Gov%20Exemplars/DOE%20examplar%20P2%202014%20MEMO.pdf

B WHAT EXPERIENCE AND RESEARCH TELLS US ABOUT PREPARING FOR EXAMINATIONS

WHAT IS THE BEST WAY TO REVISE FOR A MATHS EXAM?

- Learn your theory.
- Do practice questions.
- Check your answers.
- Focus on what you can do, as well as what you can't do.
- Discuss questions and methods with fellow learners. Explain to each other – this is an excellent way to consolidate your own understanding.
- Draw up a revision plan and stick to it.

KEEP YOUR EYE ON THE PRIZE - SHARE THESE TIPS WITH YOUR LEARNERS

Revising for mathematics exams can be hard work – it can mean making sacrifices where you choose to prioritise revision over other things. Therefore, it is important to keep your eye on the prize. Think about what your maths qualification will mean for your future life and career. Hopefully this will keep you motivated when times are tough during revision.

ASSESSMENT

C

- CAPS formal assessment requirements for Term 4:
  - Final examinations (Paper 1 and Paper 2)
- The final NSC examinations will be made up as follows:

Paper 1

	Mark allocation
Algebra and Equations (and Inequalities)	(25±3)
Patterns and Sequences	(25±3)
Finance, Growth and decay	(15±3)
Functions and Graphs	(35±3)
Differential Calculus	(35±3)
Probability	(15±3)

Paper 2

	Mark allocation
Statistics	(20±3)
Analytical Geometry	(40±3)
Trigonometry	(40±3)
Euclidean Geometry and Measurement	(50±3)

TERM 4

REVISION - WEEK 1

A POLICY AND OUTCOMES

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners will have: ● worked through summaries of all Paper 1 topics ● completed a full Paper 1 in class with their teacher.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the summaries of Paper 1
   - work through the examination and teaching notes.
3. The notes and examination are available in the Resource Pack (Resources 1 and 2) for photocopying if possible.
4. Write work on the chalkboard before the learners arrive to ensure no time is wasted.

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Support learners as they consolidate all that they have learned this year.
2. Learners need to have time to ask questions and become confident in their ability to write their final examination.

DIRECT INSTRUCTION

1. Start the lesson by handing out the summary notes for Paper 1 topics available in the Resource Pack (Resource 1).
2. Work through the summary notes with learners. This should take at least an hour.
3. As you go through each topic, ask questions to ascertain how much learners remember.
4. Encourage learners to add their own notes to the summary notes you have given them.
5. Once you have revised each section (for Paper 1), hand out the past examination paper (Resource 2 in the Resource Pack – Paper 1, 2017). Work though each question in detail. Some learners may be sufficiently confident to work on their own, while others may prefer to work with you.
6. As you go through each question, give learners the opportunity to contribute and ask questions.
7. Encourage learners to use their summary notes – for answering questions and to add their own notes as they go along.

ALGEBRA AND EQUATIONS (AND INEQUALITIES)

a) Solve for x:
   (i) $x^2 + 9x + 14 = 0$
   (ii) $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places)
   (ii) $\sqrt{(x^2 - 5)} = 2x$

b) Solve for x and y simultaneously:
   $3x - y = 4$ and $x^2 + 2xy - y^2 = -2$

c) Given $f(x) = x^2 + 8x + 16$
   (i) Solve for x if $f(x) > 0$
   (ii) For which values of p will $f(x) = p$ have TWO unequal negative roots?

Teaching notes:

a)

(i) A straightforward quadratic equation where factorising is required.

(ii) Say: *The instruction regarding the number of decimal places is a clue that the quadratic formula will be used.*

(iii) Ask: *What are the steps to solving a question with surds?*
(Isolate the surd, square both sides to eliminate the surd, solve the rest of the equation – usually a quadratic – as usual).
Ask: *What must we always do at the end of solving an equation with surds?*
(Check that the solution(s) are valid).

b)
Learners should be familiar with simultaneous equations.
Ask: *What steps do we follow to solve simultaneously?*
(Isolate one variable in any equation (the simplest), use this information to substitute into the 2nd equation and solve for the unknown variable, use this solution to solve for the 2nd variable).
Remind learners that when solving simultaneous equations, they are finding the points of intersection of the functions.

c)
(i) Ask: *How do we solve quadratic inequalities?*
(Factorise, find the critical values and sketch the quadratic function on a number line. Use the diagram to establish the values of x that will make the inequality true).
Note: Remember that there are other methods of doing this– use the method your learners are familiar with).
(ii) The sketch from (i) with an added line representing $y = p$ (that could be anywhere) will assist learners to see what the value of p should be to ensure two negative roots (x-values).

Solutions:

a)

(i) $x^2 + 9x + 14 = 0$

$(x + 7)(x + 2) = 0$

$x = -7$ or $x = -2$

(ii) $4x^2 + 9x - 3 = 0$

$a = 4 \quad b = 9 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4(4)(-3)}}{2(4)}$$

$x = 0{,}29$ or $x = -2{,}54$

(iii) $\sqrt{(x^2 - 5)} = 2\sqrt{x}$

$x^2 - 5 = 4x$

$x^2 - 4x - 5 = 0$

$(x + 1)(x - 5) = 0$

$x = -1$ or $x = 5$

$\therefore x = 5$

b) $3x - y = 4$ $\qquad$ $x^2 + 2xy - y^2 = -2$

$3x - 4 = y$ $\qquad$ $\therefore x^2 + 2x(3x - 4) - (3x - 4)^2 = -2$

$x^2 + 6x^2 - 8x - (9x^2 - 24x + 16) = -2$

$x^2 + 6x^2 - 8x - 9x^2 + 24x - 16 = -2$

$-2x^2 + 16x - 14 = 0$

$-2(x^2 - 8x + 7) = 0$

$(x - 7)(x - 1) = 0$

$\therefore x = 7$ or $x = 1$

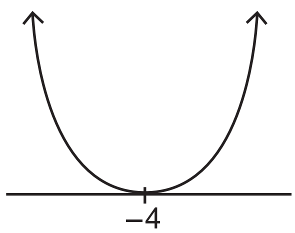
$\therefore 3(7) - 4 = y$ $\qquad$ $3(1) - 4 = y$

$17 = y$ $\qquad$ $-1 = y$

c) (i) $x^2 + 8x + 16 > 0$

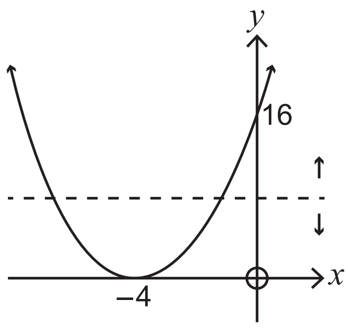
$(x + 4)(x + 4) > 0$

CV's: -4



$x \in R$; $x \neq -4$

(ii) y-Intercept: (0;16)



$0 < p < 16$

PATTERNS AND SEQUENCES

a) Given the following quadratic number pattern: 5 ; -4 ; -19 ; -40 ; …
   (i) Determine the constant second difference of the sequence.
   (ii) Determine the nth term (T_n) of the pattern.
   (iii) Which term of the pattern will equal to -25 939?

b) The first three terms of an arithmetic sequence are $2k - 7$; $k + 8$ and $2k - 1$
   (i) Calculate the value of the 15th term of the sequence.
   (ii) Calculate the sum of the first 30 even terms of the sequence.

Teaching notes:

a)
   (i) Say: *Use the method of showing the first line of difference and then the second line of difference to find the solution.*
   (ii) Note: There is more than one method to find the general term of a quadratic pattern. Use the method your learners are familiar with.
   (iii) Ask: *How can we find the position of a term?*
   (Make the term equal to the general term and solve for n).

b)
   (i) Ask: *What can be said about an arithmetic (linear) sequence?*
   (There is a common difference. $T_2 - T_1 = T_3 - T_2$).
   Say: *Use this fact to find the value of k and hence the terms in the sequence.*
   Ask: *What do we need to find T_{15}?*
   (The general term).
   (ii) Ask learners to write down the first few terms of the 'new' sequence.
   Ask: *What do we need to find the sum of terms in a sequence?*
   (The first term and the common difference as well as the number of terms being added).
   Remind learners that the formula required is on the formula sheet.

a) (i)

5   -4   -19   -40

-9   -15   -21

-6   -6

Second difference: -6

(ii)

$2a = -6$

$a = -3$

$3a + b = -9$

$3(-3) + b = -9$

$9 + b = -9$

$b = 0$

$a + b + c = 5$

$-3 + 0 + c = 5$

$c = 8$

$\therefore T_n = -3n^2 + 8$

(iii) $-3n^2 + 8 = -25\ 939$

$-3n^2 = -25\ 947$

$n^2 = 8\ 649$

$n = 93$ or $n = -93$

$\therefore n = 93$ $\quad$ -25 939 is the 93rd term

b) (i) $k + 8 - (2k - 7) = 2k - 1 - (k + 8)$

$k + 8 - 2k + 7 = 2k - 1 - k - 8$

$-k + 15 = k - 9$

$-2k = -24$

$k = 12$

$\therefore$ the pattern is 17 ; 20 ; 23 ..

$T_n = a + (n - 1)d$

$T_{15} = 17 + (15 - 1)(3)$

$T_{15} = 59$

(ii) New sequence: 20 ; 26 ; 32 ; 38…

$a = 20 \qquad d = 6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}\ (2(20) + (29)(6))$

$S_{30} = 3\ 210$

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and nth term T_n, such that

$$\sum_{n=3}^{\infty} T_n = \frac{1}{4}$$

a) If $T_1 + T_2 = 2$, write down an expression for a in terms of r.

b) Calculate the values of a and r.

Teaching notes:

Ask: *What does 'convergent' mean?*

(A series of numbers where the sum tends to a limit. The common ratio must be between -1 and 1).

a)

Ask: *What must be done to T_1 to get T_2?*

(Multiply by the common ratio).

What is the common ratio?

(r)

Say: *Make the sum of these two terms equal to 2 and solve for a*

b)

Say: *Remember that if two values are unknown, then we must have two pieces of information. It is likely that simultaneous equations will be used.*

Ask: *What are the two pieces of information given regarding a and r?*

(We know that the sum to infinity from T_3 is equal to $\frac{1}{4}$ and that the sum of the first two terms is equal to 2. In other words, the sum to infinity of the sequence is 2 + $\frac{1}{4}$. We also know the formula for S_∞).

a) $T_1 = a \qquad T_2 = ar$

$\therefore a + ar = 2$

$a(1 + r) = 2$

$\therefore a = \dfrac{2}{1+r}$

b)

$$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n = \frac{1}{4}$$

$$= 2 + \frac{1}{4}$$

$$= \frac{9}{4}$$

$$S_\infty = \frac{a}{1-r}$$

$$\therefore \frac{a}{1-r} = \frac{9}{4}$$

$$4a = 9 - 9r$$

$$9r = 9 - 4a$$

$$r = \frac{9-4a}{9}$$

$$a + ar = 2 \quad \text{(from a))}$$

$$a + a\frac{9-4a}{9} = 2$$

$$9a + a(9 - 4a) = 18$$

$$9a + 9a - 4a^2 = 18$$

$$-4a^2 + 18a - 18 = 0$$

$$2a^2 - 9a + 9 = 0$$

$$(2a - 3)(a - 3) = 0$$

$$a = \frac{3}{2} \quad \text{or} \quad a = 3$$

$$\therefore r = \frac{1}{3} \quad \text{or} \quad r = -\frac{1}{3}$$

N/A (only positive terms)

FINANCE, GROWTH AND DECAY

a) Mbali invested R10 000 for 3 years at an interest rate of r % p.a. compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place.

b) Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a. compounded monthly.

  (i) Calculate Piet's monthly instalment.

  (ii) Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan.

Teaching notes:

a)

Ask: *Are regular payments mentioned in this situation?*

(No).

What formula will we use?

(The compound interest formula).

Remind learners to be careful at the end when converting the decimal into a percentage.

b)

(i) Ask: *Are regular payments mentioned in this situation?*

(Yes).

Ask: *What formula will we use?*

(Present value as it is a loan).

(ii) Discuss this question in detail with learners. The need to have understanding of several concepts.

Tell learners that it would be useful to find the balance outstanding. From this we can calculate the difference between this amount and the original loan to find exactly how much he has paid so far towards the payment of the loan.

Once this amount has been found, we need to calculate what he has paid so far.

The difference between these amounts is the extra that he has paid – in other words the interest.

Ask: *How do we find balance outstanding?*

(We use the present value formula and the monthly instalment and the months still to go for the value of n).

Solutions:

a)
$$A = P(1 + i)^n$$
$$12\,146{,}72 = 10\,000\left(1+\frac{i}{12}\right)^{3\times 12}$$
$$1{,}214672 = \left(1+\frac{i}{12}\right)^{36}$$
$$\sqrt[36]{1{,}214672} = \left(1+\frac{i}{12}\right)$$
$$\sqrt[36]{1{,}214672}-1 = \frac{i}{12}$$
$$0{,}0054166\ldots = \frac{i}{12}$$
$$\therefore r = 0{,}54166 \times 12 \times 100 = 6{,}5\%$$

b) (i)
$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$
$$235\,000 = \frac{x\left[1-(1+\frac{0{,}11}{12})^{-54}\right]}{\frac{0{,}11}{12}}$$
$$\frac{(\frac{0.11}{12}\times 235\,000)}{\left[1-(1+\frac{0.11}{12})^{-54}\right]} = x$$
$$x = \text{R}5\,536{,}95$$

(ii) Balance outstanding:
$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$
$$P = \frac{5\,536{,}95\left[1-(1+\frac{0{,}11}{12})^{-42}\right]}{\frac{0{,}11}{12}}$$
$$P = 192\,296{,}20$$

Loan amount - balance outstanding

= 235 000 – 192 296,20

= 42 703,80

Total paid so far:

R5 536,95 × 12 = R66 443,40

Interest paid:

R66 443,40 – 42 703,80 = R23 739,60

FUNCTIONS AND GRAPHS

Given $f(x) = -ax^2 + bx + c$

a) The gradient of the tangent to the graph of f at the point $(-1; \frac{7}{2})$ is 3. Show that $a = \frac{1}{2}$ and $b = 2$

b) Calculate the x-intercepts of f.

c) Calculate the cooordinates of the turning pont of f.

d) Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point.

e) Use the graph to determmine the values of x for which $f(x) > 6$

f) Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes.

g) Write down the values of x for which $f(x).g(x) \leq 0$

Teaching notes:

a)

Remind learners that if two variables are unknown, we must have two pieces of information.

Ask: *What does 'gradient of the tangent' tell us?*

(That the derivative of the function is equal to 3 when $x = -1$).

Say: *Find the derivative, make it equal to 3 and substitute* $x = -1$.

Ask: *What is the second piece of information?*

(We know that the point $(-1; \frac{7}{2})$ lies on the function).

Say: *Use this to find the a 2nd statement. Solve simultaneously for a and b.*

Remind learners that the values given for a and b may NOT be used in the calculations. These values can, however, be used as a check.

b)

Point out that the information regarding a and b in a) can be used in b) even if a) may have not been answered correctly.

Ask: *How do we find the x-intercepts of any function?*

(Make the function equal to zero and solve for x).

c)

Ask: *How do we find the turning point of a function?*

(Find the derivative, make it equal to zero and solve for the x-value.

Substitute this into the function to find the corresponding y-value).

Ask: *Why do we make the derivative equal to zero?*

(The gradient of a tangent will be zero at a turning point as it is a horizontal line).

Note: Learners can use other methods to find turning point. The above method is recommended so that learners can revise a few concepts from Calculus).

Remind learners that the turning point of a quadratic function is always halfway between the two x-intercepts; in other words, it lies on the axis of symmetry.

d)

Learners should be able to draw the function with the above information. Point out that the question requires that all intercepts and turning points to be clearly marked.

e)

Ask learners to draw in the line y = 6 and highlight the part of the function that is greater than (above) this line.

Ask: *What are the corresponding x-values for the highlighted part of the function?*

(The second x-value can be found through symmetry or by making the function equal to 6 and solving).

f)

Learners should be able to draw a straight-line graph with ease.

g)

Ask: *What sign should integers be for the product to be less than zero?*

(One should be positive, and one should be negative).

Remind learners that ≤ will mean that the critical values should be included.

Tell learners to run their ruler from left to right, parallel to the y-axis. Learners should look for areas where one function is negative while the other function is positive.

This will give the x-values to answer the question.

Note: Many learners find this challenging – ensure you show them the method on the chalkboard after they have had an opportunity to try it themselves.

a)

$$f(x) = -ax^2 + bx + 6$$
$$f'(x) = -2ax + b$$
$$f'(-1) = 3$$
$$-2a(-1) + b = 3$$
$$2a + b = 3$$
$$f(x) = -ax^2 + bx + 6$$
$$f(-1) = -a(-1)^2 + b(-1) + 6$$
$$\tfrac{7}{2} = -a - b + 6$$
$$7 = -2a - 2b + 12$$

From: $2a + b = 3$

$$b = 3 - 2a$$
$$7 = -2a - 2(3 - 2a) + 12$$
$$7 = -2a - 6 + 4a + 12$$
$$1 = 2a$$
$$a = \tfrac{1}{2}$$
$$b = 3 - 2\left(\tfrac{1}{2}\right) = 2$$

b) $f(x) = -\frac{1}{2}x^2 + 2x + 6$

$0 = -\frac{1}{2}x^2 + 2x + 6$

$0 = x^2 - 4x - 12$

$0 = (x - 6)(x + 2)$

$x = 6$ or $x = -2$

(6;0) (-2;0)

c) $f'(x) = -x + 2$

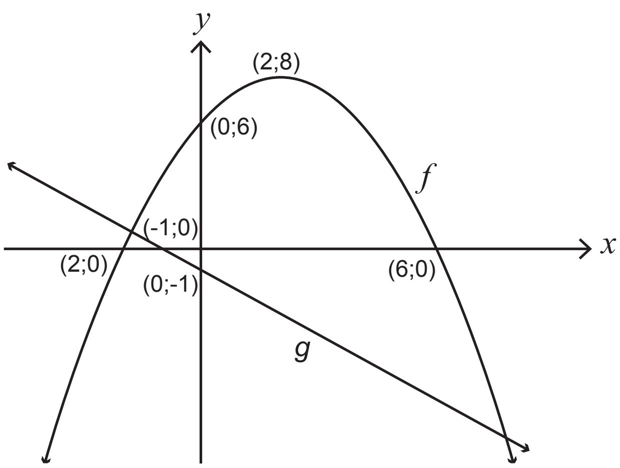
$0 = -x + 2$

$x = 2$

$f(2) = -\frac{1}{2}(2)^2 + 2(2) + 6 = 8$

T Pt: (2;8)

d) and f)



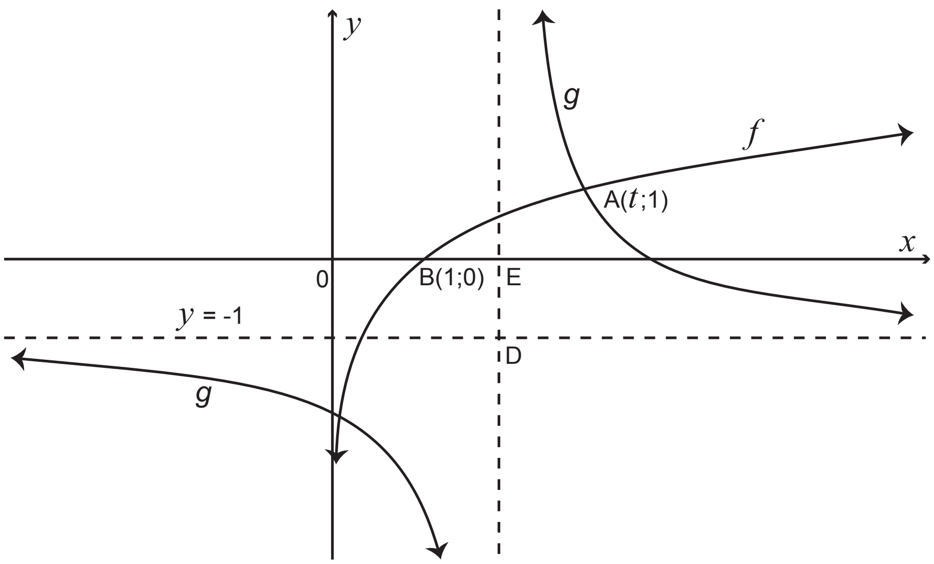
e) $0 < x < 4$

f) above

g) $x \leq -2$ or $-1 \leq x \leq 6$

The diagram below shows the graphs of $g(x) = \frac{2}{x+p} + q$ and $f(x) = log_3 x$

- y = -1 is the horizontal asymtote of g
- $B(1;0)$ is the x-intercept of f
- $A(t;1)$ is a point of intersection between f and g
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- $OB = BE$



a) Write down the range of *g*.
b) Determine the equation of *g*.
c) Calculate the value of t.
d) Write down the equation of $f^{-1}(x)$, the inverse of f, in the form $y = \cdots$
e) For which values of x will $f^{-1}(x) < 3$.
f) Determine the point of intersection of the graphs of f and the axis of symmetry of *g* that has a negative gradient.

Teaching notes:

a)

Ask: *What does 'range' mean?*

(All the possible y-values for the function).

How do we find the range?

(Run the ruler from the bottom to the top of the graph, holding it horizontally. Focus on what y-values are being used for the function).

b)
Ask: *What is the coordinate at E?*
(2;0).
Remind learners that no information is given that isn't useful. In this case, $OB = BE$.
Tell learners that they now have the equations of both asymptotes and should be able to substitute these values into the appropriate places in the equation.

c)
Ask: *How can we find the x-coordinate of point A?*
(Substitute into one of the functions).
Point out that any function will work but the log function will be the quicker calculation and it is also a function that was given. As learners had to find the function of the hyperbola remind them that they could have made an error and will therefore be using the error to find t.
d)
Ask: *What is an inverse function?*
(A function that is reflected in the line $y = x$ and therefore the rule applied is to interchange the x and y coordinates).
Point out that learners require a knowledge of logarithms.
e)
Tell learners that they will need to make an inequality to find the value(s) of x where the exponential function is less than 3.
f)
Say: *First we need the equation of the axis of symmetry.*
Ask: *What can you tell me about the axes of symmetry of the hyperbola?*
(One has a gradient of 1 while the other has a gradient of -1.
Both pass through the point where the asymptotes meet).
Say: *As the question mentions a negative gradient, the line we are looking for has a gradient of* -1 *and passes through the point* (2;-1).
Ask: *Once we have the equation of the axis of symmetry, what will we need to do next?*
(Find the point of intersection).
Point out that this can be done by inspection.

a) $y \in R$; $y \neq -1$

b) $g(x) = \frac{2}{x-2} - 1$

c) $f(x) = log_3 x$

$1 = log_3 t$

$\therefore t = 3$

d) $x = log_3 y$

$y = 3^x$

e) $3x < 3$

$x < 1$

f) Axis of symmetry:

$m = -1$ and y-intercept: $(0;1)$

$y = -x + 1$

x-intercept of axis of symmetry: $(1;0)$

$\therefore$ point of intersection is $(1;0)$

DIFFERENTIAL CALCULUS

a) Given $f(x) = 2x^2 - x$. Determine $f'(x)$ from first principles.

b) Determine:

(i) $D_x\,[(x + 1)(3x - 7)]$

(ii) $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}\pi$

Teaching notes:

a)

Remind learners that there is almost always a question in which they are required to find the derivative from first principles. Tell learners that no marks will be awarded for using the rules (shortcut). They must practise finding the derivative from first principles to ensure they get the maximum marks.

Point out that learners must be very careful with their setting out – the limit as h tends to zero must remain in the calculation until $h = 0$ is substituted.

Even though the rules may not be used, learners should be encouraged to use this method to check their answer.

b)

Remind learners:

There can be no:

- brackets in the function (multiplying out and collecting like terms is necessary)
- surds in the function (surd rules will be used to change into exponential form)
- variables in the denominator position (exponential rules will be used to ensure all variables are in the numerator position).

These must be dealt with before any rules can be performed to find the derivatives.

a)
$$f(x) = 2x^2 - x$$
$$f(x+h) = 2(x+h)^2 - (x+h)$$
$$f(x+h) = 2(x^2 + 2xh + h^2) - x - h$$
$$f(x+h) = 2x^2 + 4xh + 2h^2 - x - h$$
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - x - h(2x^2 - x)}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{4xh + 2h^2 - h}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{h(4x + 2h - 1)}{h}$$
$$f'(x) = \lim_{h\to 0} 4x + 2h - 1$$
$$f'(x) = 4x - 1$$

b) (i) $D_x[(x + 1)(3x - 7)]$
$= D_x[3x^2 - 4x - 7]$
$= 6x - 4$

(ii)
$$y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}\pi$$
$$y = x^{\frac{3}{2}} - 5x^{-1} + \frac{\pi}{2}$$
$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-2}$$
$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-2}$$

Given $f(x) = x(x - 3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

a) Show that f has a point of inflection at $x = 2$.

b) Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points.

c) For which values of x will $y = -f(x)$ be concave down?

d) Use your graph to answer the following questions:

(i) Determine the coordinates of the local maximum of h if $h(x) = f(x - 2) + 3$

(ii) Claire claims that $f'(2) = 1$. Do you agree with Claire? Justify your answer.

Teaching notes:

a)

Ask: *What is the point of inflection?*

(Point where a cubic graph changes from concave up to concave down or vice versa).

How do we find the point of inflection?

(Find the second derivative, make it equal to zero and solve for x).

b)

Ask: What can we deduce from $f(x) = x(x - 3)^2$?

(The cubic function is positive, so it increases at the beginning; the repeated factor tells us that there is a turning point and an x-intercept where $x = 3$)

What does $f'(1) = f'(3) = 0$ *tell us?*

(That when $x = 1$ and $x = 3$, the derivative is equal to zero and there is therefore a turning point on the function at these x-values).

What does $f(1) = 4$ *tell us?*

(That the point (1;4) lies on the function).

c)

Note: First discuss with learners the concavity of the function given.

(That it is concave down for $x < 2$ and concave up for $x > 2$).

Ask: *What do we already know about where the function is concave down or concave up?*

(The point of inflection is at $x = 2$).

If a function is transformed to become $-f(x)$*, what was the transformation?*

(It has been reflected in the x-axis).

In this case, what will happen to the x*-value at the point of inflection?*

(Nothing – it will remain $x = 2$).

How will the concavity of the new function relate to the original function?

(It will be the opposite – where the function became concave up, the new function will become concave down).

d)

(i) Ask: *What transformations have been done to the function if* $f(x - 2) + 3$?

(Horizontal shift of 2 units to the right and a vertical shift of 3 units up).

Which point is the local maximum of the original function?

(1;4)

What will the new point be after the transformation?

(3;7)

(ii) Tell learners that they could answer this by looking at the function and checking if it is increasing (positive gradient) or decreasing (negative gradient) at $x = 1$.

Learners could also find the value of $f'(-1)$.

a) $f(x) = x(x^2 - 6x + 9)$

$f(x) = x^3 - 6x^2 + 9x$

$f'(x) = 3x^2 - 12x + 9$

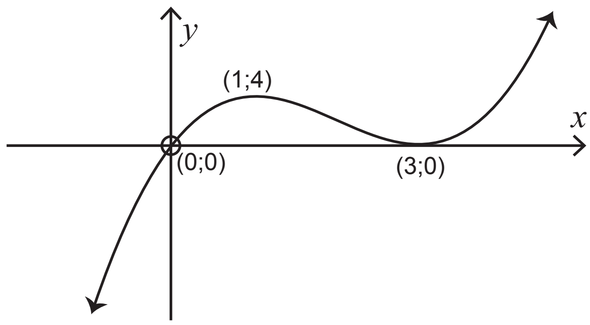
$f''(x) = 6x - 12$

$0 = 6x - 12$

$12 = 6x$

$2 = x$

b)



c) $x > 2$

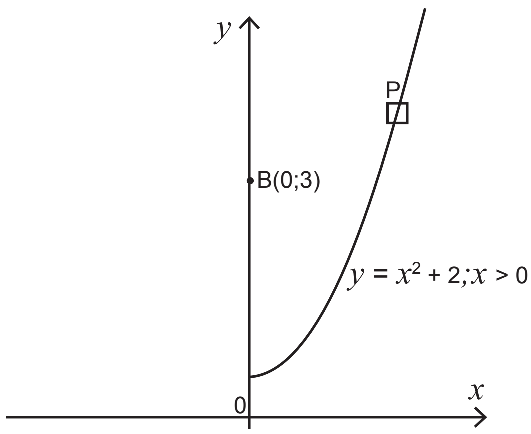
d) (i) (3;7)

(ii) Do not agree. The function is decreasing when $x = 2$ so it cannot be positive.

$f'(2) = 3(2)^2 - 12(2) + 9 \neq 1$

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes are chosen as shown in the diagram.

Benny sits at a vantage point $B(0;3)$ and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny.

Teaching notes:

Tell learners that 'the closest' should give them a clue that they are dealing with a maximum/ minimum question.

Ask: *What is the coordinate of P in terms of x?*

$(x; x^2 + 2)$.

How can we find the distance PB?

(By using the distance formula).

How can we use this to find the minimum value?

(Find the derivative, make it equal to zero and solve for x).

Tell learners that they will use this value to find the corresponding y-value and find the minimum.

Solutions:

The distance PB must be at a minimum.

$B(0;3)$ $\quad P(x;x^2 + 2)$

$$PB^2 = (x - 0)^2 + (x^2 + 2 - 3)^2$$
$$PB^2 = (x - 0)^2 + (x^2 - 1)^2$$
$$PB^2 = x^2 + x^4 - 2x^2 + 1$$
$$PB^2 = x^4 - x^2 + 1$$
$$\frac{d(PB^2)}{dx} = 4x^3 - 2x$$
$$0 = 4x^3 - 2x$$
$$0 = 2x(2x^2 - 1)$$
$$2x = 0 \qquad 2x^2 - 1 = 0$$
$$x = 0 \qquad 2x^2 = 1$$
$$x^2 = \frac{1}{2}$$
$$x = \frac{1}{\sqrt{2}}$$
$$\therefore PB^2 = x^4 - x^2 + 1$$
$$PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$$
$$= \frac{3}{4}$$
$$\therefore PB = \frac{\sqrt{3}}{2}$$

Closest distance: $\frac{\sqrt{3}}{2} = 0{,}87$

PROBABILITY

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp(W) on their cell phones. The survey revealed the following:

- 8 use all three
- 12 use Instagram and Twitter
- 5 use Twitter and WhatsApp, but not Instagram
- x use Instagram and WhatsApp, but not Twitter
- 61 use Instagram
- 19 use Twitter
- 73 use WhatsApp
- 14 use none of these applications.

a) Draw a Venn diagram to illustrate the information above.

b) Caculate the value of x.

c) Calculate the probability that a learner, chosen randomly, uses only ONE of these applications.

Teaching notes:

a)

Ask: *How many sets are represented in the information?*

(3).

Is there an intersection? (Yes)

Say: *Draw three circles that intersect and label them according to the given headings.*

Ask: *Where should we always start when completing a Venn diagram?*

(The intersection).

How many learners use all three applications?

(8)

Tell learners to complete the Venn diagram themselves before doing it on the board with them. Explain each step (and the subtraction essential) as you complete each section.

b)

Ask: *What knowledge will we use to find the value of x?*

(Knowing that 100 learners make up the sample space and that all the totals used in the Venn diagram should total 100. Make an equation and solve.).

c)

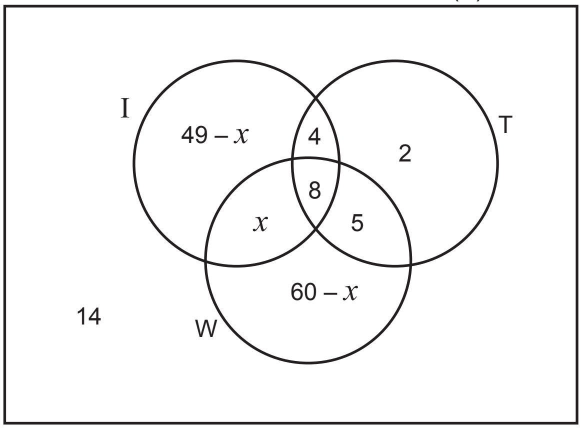
Ask a learners to come to the board and show which areas represent the use of only one application.

($49 - x$; 2 ; $60 - x$)

Remind learners that because this is a probability question the sample space is important.

Solutions:

a)



b) $49 - x + 4 + 2 + x + 8 + 5 + 60 - x + 14 = 100$

$-x + 142 = 100$

$x = 42$

c) P(use only one) $= \frac{(7+2+18)}{100}$

$= \frac{27}{100} = 0{,}27$

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR45789

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

a) How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits?

b) Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system.

Teaching notes:

a)

Tell learners to use the 'dash method' and write in all the possibilities available for each space before using the fundamental counting principle and multiplying the possibilities.

b)

Tell learners that the answer in a) will assist them to extend the number of digits until the number of possibilities exceeds 700 000.

a) __5__ __5__ __10__ __9__

$5 \times 5 \times 10 \times 9 = 2\ 250$

b) 2 letters, 3 digits: $5 \times 5 \times 10 \times 9 \times 8 = 18\ 000$

2 letters, 4 digits: $5 \times 5 \times 10 \times 9 \times 8 \times 7 = 126\ 000$

2 letters, 5 digits: $5 \times 5 \times 10 \times 9 \times 8 \times 7 \times 6 = 756\ 000$

2 letters and 5 digits are required to ensure unique numbers for 700 000 clients

8. When the past paper has been completed ask learners if they have any questions.
9. Say: *Next week we will be revising the work for Paper 2.*

TERM 4

REVISION - WEEK 2

A POLICY AND OUTCOMES

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners will have: ● worked through summaries of all Paper 2 topics ● completed a full Paper 2 in class with the support of their teacher.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the summaries of Paper 2
   - work through the examination and teaching notes.
3. The notes and examination are available in the Resource Pack (Resources 3 and 4) for photocopying if possible.
4. Write work on the chalkboard before the learners arrive to ensure no time is wasted.

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Support learners as they consolidate all that they have learned this year.
2. Learners need to have time to ask questions and become confident in their ability to write their final examination.

DIRECT INSTRUCTION

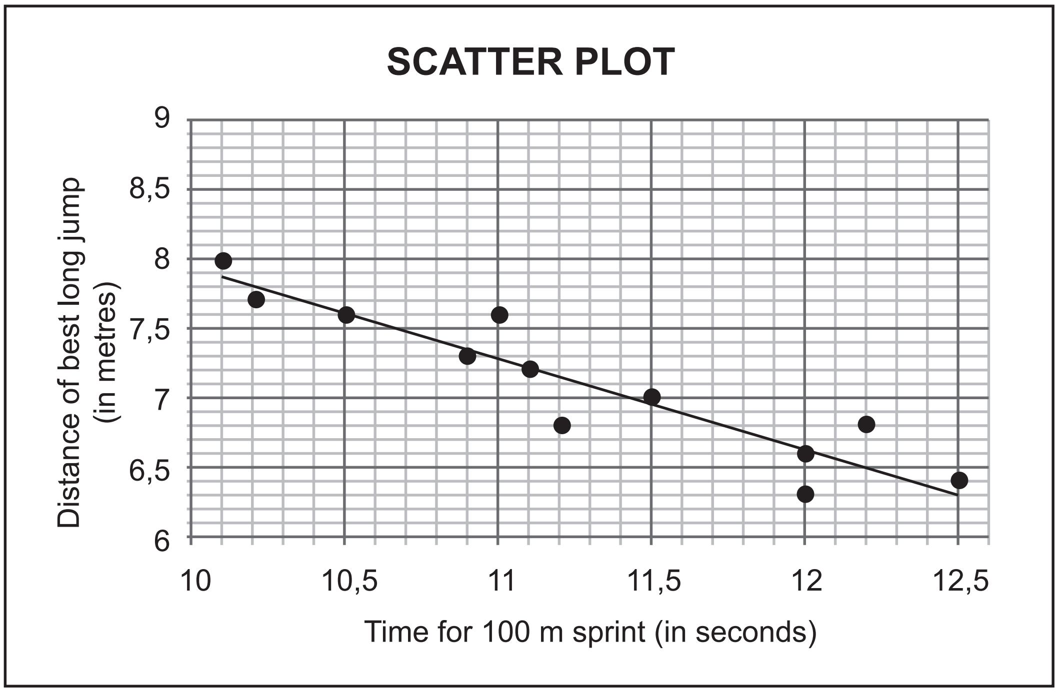
1. Start the lesson by handing out the 4 sets of summary notes for Paper 2 topics available in the Resource Pack (Resource 3).
2. Go through the notes with learners. This should take at least an hour.
3. Ask questions to ascertain how much learners remember as you go through each topic.
4. Encourage learners to add their own notes to the summaries – now and throughout the next few weeks of revision.
5. Once each section has been covered, hand out the past examination paper (Paper 2 2017) (Resource 4). Do each question in detail with learners. Allow learners who feel confident to work on their own to do so.
6. As you go through each question, give learners the opportunity to contribute and ask questions.
7. Encourage learners to refer to their summary notes and to use them when answering questions or to add notes to if they are finding something a challenge.

STATISTICS

The table below shows the time (in seconds, rounded to ONE decimal place) taken by 12 athletes to run the 100-metre sprint and the distance (in metres, rounded to ONE decimal place) of their best long jump.

Time for 100 m sprint (in seconds)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

The scatter plot representing the data above is given below.



The equation of the least squares regression line is $\hat{y} = a + bx$.

a) Determine the values of a and b.

b) An athlete runs the 100-metre sprint in 11,7 seconds. Use $\hat{y} = a + bx$ to predict the distance of the best long jump of this athlete.

c) Another athlete completes the 100-metre sprint in 12,3 seconds and the distance of his best long jump is 7,6 metres. If this is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations.

Teaching notes:

a)

Tell learners that they need to be proficient in their calculator work and know how to find the equation of the least squares regression line.

b)

Ask: *What must we do to predict this distance?*

(Substitute the sprint time into the equation found).

Note: Discuss interpolation and extrapolation while answering this question.

c)

Tell learners to plot the point given.

Ask: *How would this point affect the least squares regression line?*

(It would pull the line towards it and therefore make it steeper, increasing the gradient).

a) a = 14,34 b = -0,642

b) y = 14,34 – 0,642x

y = 14,34 – 0,642(11,7)

y = 6,85

c) The gradient will increase. The new point lies well above the current set of data.

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

a) Calculate:
   (i) The mean of the data
   (ii) The interquartile range of the data

b) The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours?

c) Draw a box and whisker diagram to represent the data on the number line provided.

d) The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15;21;23,5;26;38)
   (i) Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles?
   (ii) The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prize winners? Motivate your answer.

Teaching notes:

a)
   (i) Learners should be proficient at finding the mean and not require assistance.
   (ii) Ask: *What do we need to find the interquartile range?*
   (Lower quartile and upper quartile).
   How do we find these?
   (With a small set of data, we could count to find the position of the lower and upper quartile. Alternately we can use $\frac{1}{4}(n+1)$ and $\frac{3}{4}(n+1)$ to find the position of each).

b)

Remind learners what standard deviation means and how to find information that lies within one (or two) standard deviations from the mean.

Say: *Add the standard deviation to the mean to find the time required to answer the question.*

Tell learners to note that we are only adding the standard deviation and not subtracting it because of the way the question has been asked. This question is only linked to the girls that took longer. Remind learners that the standard deviation is usually added AND subtracted to find all values within one standard deviation from the mean.

c)

Ask: *What numbers do we need to draw a box and whisker diagram?*

(Lowest value, lower quartile, median, upper quartile, highest value).

Remind learners that the diagram must be drawn to scale.

d)

(i) This should be straightforward for learners to answer – they need only compare the two medians.

(ii) Ask: *How many girls were faster than the fastest boy?*

(5)

How many boys will get a prize?

(None).

a) (i) $\overline{x} = \frac{472}{23} = 20{,}52$ seconds

(ii) $Q_1 = 16$

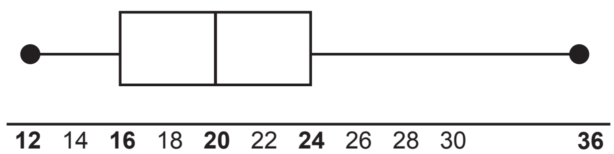
$Q_3 = 24$

$IQR = 24 - 16 = 8$

b) $20{,}52 + 5{,}94 = 26{,}46$

$\therefore$ 4 girls have a time longer than 26,46

c)



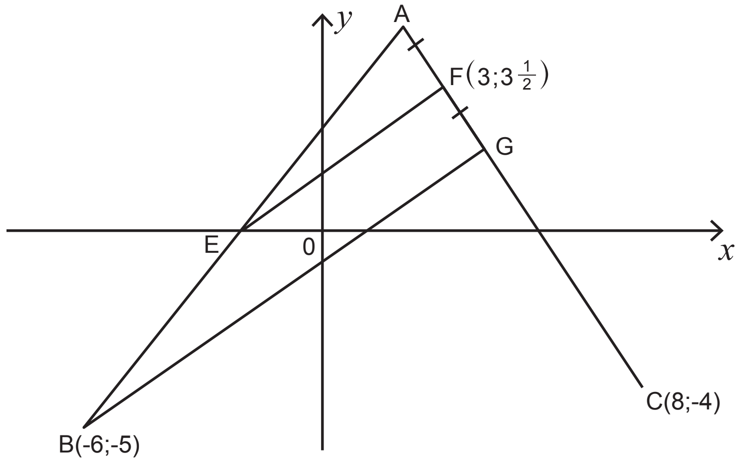
d) (i) Girls

(ii) Best time for boys: 15 seconds. 5 girls were faster than this.

$\therefore$ no boys will receive a prize.

ANALYTICAL GEOMETRY

In the diagram, A, $B(-6;-5)$ and $C(8;-4)$ are points in the Cartesian plane. $F(3;3\frac{1}{2})$ and G are the points on line AC such that $AF = FG$. E is the x-intercept of AB.



a) Calculate:
   (i) The equation of AC in the form $y = mx + c$
   (ii) The coordinates of G if the equation of BG is $7x - 10y = 8$

b) Show by calculation that the coordinates of A is (2;5)

c) Prove that $EF \parallel BG$.

d) ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D.

Teaching notes:

a)

(i) Ask: *What do we need to find the equation of any straight line?*
(The gradient and a point).

(ii) Ask: *What is happening at point G on the diagram?*
(It is where the two lines meet – a point of intersection).
How do we find a point of intersection of two straight lines?
(Solve simultaneously)

b)

Point out that when they are given the solution in a question and asked to 'prove that' or 'show that', they may NOT use the information. They should rather read the question as 'Find the coordinates of A' and use what is given as a check.

Ask: *What knowledge do we need to use to find the coordinates of A?*
(Midpoint of a line segment. It will be used in reverse, as we are given the midpoint and need one of the outer points).

c)
Ask learners what theorem may be used to answer this question. (Visually, they need to recognise the midpoint theorem within ΔABG)
Tell learners to find the midpoint of AB.
Ask: *What do you notice?*
(It is the x-intercept of the line AB).
d)
Remind learners that the parallelogram is given in order – in other words, to sketch it would mean that it must go from A, to B, to C and then D. This means that D could only be in quadrant 1 – top right.
Since $AB \parallel CD$, ask: *what do we need to do to get from B to A?* (up 10 right 8)
Ask: *What will the coordinate D be if we make the same shift from C?*

a) (i)
$$m = \frac{-4-3\frac{1}{2}}{8-3}$$
$$m = -\frac{3}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -\frac{3}{2}(x - 8)$$
$$y = -\frac{3}{2}x + 12 - 4$$
$$y = -\frac{3}{2}x + 8$$

(ii)
$$y = -\frac{3}{2}x + 8 \qquad 7x - 10y = 8$$
$$7x - 10(-\frac{3}{2}x + 8) = 8$$
$$7x + 15x - 80 = 8$$
$$22x = 88$$
$$x = 4$$
$$y = -\frac{3}{2}(4)+8$$
$$y = 2 \qquad \therefore G(4;2)$$

b) $A(x;y)$ $\quad F(3;3\frac{1}{2})$ $\quad G(4;2)$
$$\frac{x+4}{2} = 3 \qquad \frac{y+2}{2} = \frac{7}{2}$$
$$x + 4 = 6 \qquad y + 2 = 7$$
$$x = 2 \qquad y = 5$$
$\therefore A(2;5)$

c) Midpoint AB: $\left(\frac{-6+2}{2};\frac{-5+5}{2}\right)$

$= (-2;0)$

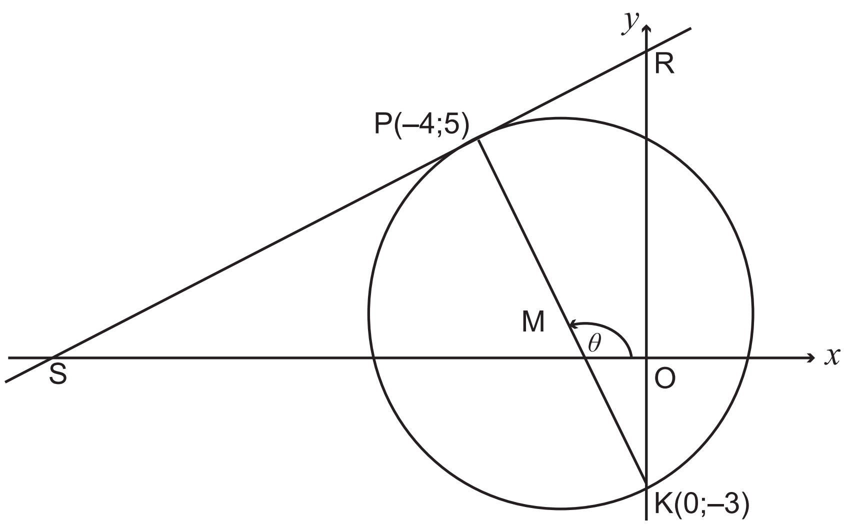
E is the x-intercept of AB (given)

$\therefore E$ is the midpoint of AB

$\therefore EF \parallel BG$ (midpoint theorem)

d) $D(16;6)$ (by inspection)

In the diagram, $P(-4;5)$ and $K(0;-3)$ are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



a) Determine:
   (i) The gradient of SR
   (ii) The equation of SR in the form $y = mx + c$
   (iii) The equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$
   (iv) The size of $P\hat{K}R$
   (v) The equation of the tangent to the circle at K in the form $y = mx + c$

b) Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points.

c) Calculate the area of ΔSMK.

Teaching notes:

(i) Ask: *What knowledge is required to find gradient as we don't have two coordinates to use the formula?*
(radius $\perp$ tangent)

(ii) Ask: *What do we need to find the equation of any straight line?*
(The gradient and a point).

(iii) Ask: *What do we need to find the equation of a circle?*

(The radius and the centre).

How can we find the centre?

(Midpoint).

How can we find the radius?

(Distance).

(iv) Remind learners that when an angle is required that is NOT an angle of inclination, there will be a need to use Grade 8 geometry.

Ask: *How we can find this angle?*

Ask a learner to come and show the class on the board.

(Find θ – angle of inclination of PK, then use the exterior angle of a triangle to find $P\hat{K}R$. This is possible due to the angle at the origin being 90º).

(v) Remind learners that the equation of a tangent is the equation of a straight line.

Ask: *What do we need to find the equation of any straight line?*

(The gradient and a point).

Point out that in this case, the point is also the y-intercept.

b)

Ask: *Which other lines have a gradient of $\frac{1}{2}$?*

(The two tangents).

Use a ruler. Show learners that, as we move from the tangent at K, keeping the ruler parallel (equal gradients), the line will cut the circle in two places from the first tangent to the second tangent.

Remind learners that t represents the y-intercept.

Ask: What are the values of t?

c)

Ask: *What are the two ways that we can find the area of a triangle?*

(Using the formula A = $\frac{1}{2}$ b.$\perp$ht or by using the area rule A = $\frac{1}{2}$ $absin$ Ĉ).

Ask learners to look carefully at the diagram and the information already found and decide which one of these two options is the most likely.

Learners may find this difficult. They need to recognise that not only will they need to use the Area rule but will also need other trigonometry knowledge to find the information needed to substitute into the Area rule.

There are a few different ways that this question could be answered. If time permits, explore more than one way to the solution.

a)

(i) $m_{PK} = \frac{5-(-3)}{-4-0}$

$= -2$

$\therefore m_{SR} = \frac{1}{2} \qquad (m_{PK} \perp m_{SR})$

(ii) $m = \frac{1}{2} \qquad (-4;5)$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{2}(x - (-4))$

$y - 5 = \frac{1}{2}x + 2$

$y = \frac{1}{2}x + 7$

(iii) Midpoint of PK:

$M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right)$

$M(-2;1)$

$MK^2 = (-2 - 0)^2 + (1 - (-3))^2$

$MK^2 = (-2)^2 + (1 + 3)^2$

$MK^2 = 4 + 16$

$MK^2 = 20$

$\therefore (x + 2)^2 + (y - 1)^2 = 20$

(iv) $\tan\theta = m$

$\tan\theta = -2$

$RA = 63{,}43°$

$\theta = 180° - 63{,}43° = 116{,}57°$

$P\hat{K}R = 116{,}57° - 90° = 26{,}57° \qquad$ (ext < of Δ)

(v) $m_{PK} = -2 \qquad \therefore m_{SR} = \frac{1}{2}$

$\therefore$ y $= \frac{1}{2}x - 3$

b) y-intercept of SR: 7

y-intercept of tangent: -3

$\therefore -3 < t < 7$

c) $S(-14;0)$ (x-intercept of SR)

By the distance formula:

$SM = \sqrt{145}$

$MK = \sqrt{20}$

$SK = \sqrt{205}$

$m^2 = k^2 + s^2 - 2ks.\cos\hat{M}$

$\sqrt{205}^2 = \sqrt{145}^2 + \sqrt{20}^2 - 2(\sqrt{145})(\sqrt{20}).\cos\hat{M}$

$\cos\hat{M} = \frac{\sqrt{205}^2 - \sqrt{145}^2 + \sqrt{20}^2}{2(\sqrt{145})(\sqrt{20})}$

$\therefore \hat{M} = 118{,}8°$

Area $\Delta SMK = \frac{1}{2}(SM)(MK)\sin\hat{M}$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20})\sin 118{,}8°$

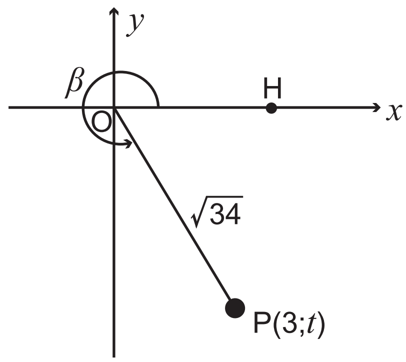
$= 24{,}9985$ square units

TRIGONOMETRY

a) Given: $\dfrac{\sin(A-360^\circ).\cos(90^\circ+A)}{\cos(90^\circ-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio.

b) In the diagram, $P(3;t)$ is a point in the Cartesian plane. $OP = \sqrt{34}$ and $H\hat{O}P = \beta$ is a reflex angle.



Without using a calculator, determine the value of:

(i) t

(ii) $\tan\beta$

(iii) $\cos 2\beta$

c) Prove:

(i) $\sin(A + B) - \sin(A - B) = 2\cos A.\cos B$

(ii) Without using a calculator, show that $\sin 77^\circ - \sin 43^\circ = \sin 17^\circ$

Teaching notes:

a)

Tell learners that they need to know their reductions. Remind them to first decide which quadrant the angle is in then decide whether the trig ratio is positive or negative in that quadrant.

Ask: *Who can remind me how we reduce angles that involve a* $90^\circ -$ *or* $90^\circ +$*?*

(Use co-ratios)

b)

(i) Ask: *What will be required to answer this question?*

(Theorem of Pythagoras)

Remind learners of the work often required before they will be able to answer the actual ratios asked.

(ii) Learners will find this simple if they know their ratios well.

(iii) Ask: *What will be required to answer this question since the angle used in the original information is* β *and not* 2β*?*

(The double angle needs to be expanded).

Ask: *Does it matter which of the three versions we use?*

(No),

c)

Remind learners that when proving an identity, they should work with one side at a time.

(i) Ask: *What knowledge is required to prove this identity?*
(Expansion of compound angles).

(ii) Remind learners that ‘without the use of a calculator’ usually implies that special angles will be required.

Ask: *How will we proceed due to these angles not being special angles?*
(Use compound angles – expand the two angles given to include a special angle).

Point out that way the questions are numbered indicates that the two parts of this question are connected in some way. We can often save time by using this information.
Show learners the connection when you reach the step in c) (ii).

a) $\dfrac{\sin(A-360^\circ).\cos(90^\circ+A)}{\cos(90^\circ-A).\tan(-A)}$

$= \dfrac{\sin A.(-\sin A)}{\sin A.(-\tan A)}$

$= \dfrac{\sin A}{\tan A}$

$= \dfrac{\sin A}{\frac{\sin A}{\cos A}}$

$= \sin A \times \dfrac{\cos A}{\sin A}$

$= \cos A$

b) (i) $3^2 + t^2 = \sqrt{34}^{\,2}$

$t^2 = 25$

$t = 5 \qquad \therefore t = -5$

(ii) $\tan\beta = \frac{-5}{3}$

(ii) $\cos 2\beta = 1 - 2sin^2\beta$

$= 1 - 2\left(\frac{-5}{\sqrt{34}}\right)^2$

$= 1 - 2\frac{25}{34}$

$= 1 - 2\frac{25}{34}$

$= -\frac{8}{17}$

c) (i) LHS = $\sin(A + B) - \sin(A - B)$

$= \sin A.\cos B + \cos A.\sin B - (\sin A.\cos B - \cos A.\sin B)$

$= \sin A.\cos B + \cos A.\sin B - \sin A.\cos B + \cos A.\sin B$

$= 2\cos A.\sin B$

$= RHS$

(ii) $\sin 77° - \sin 43°$

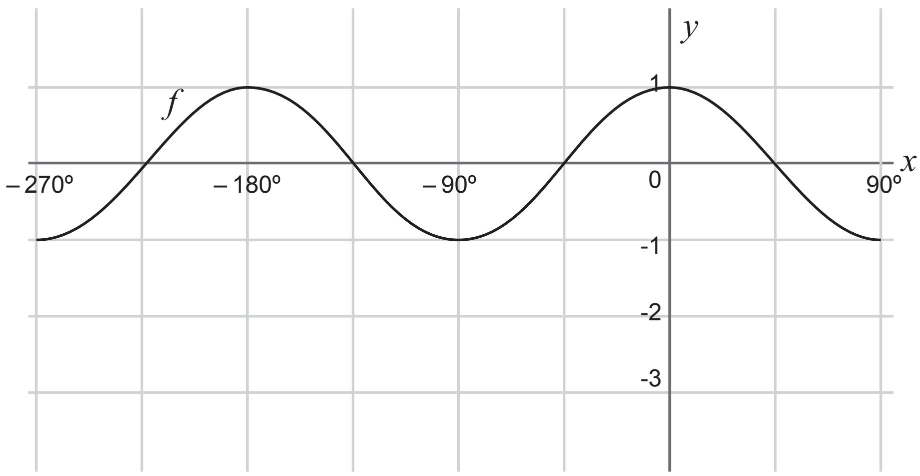
$= \sin(60° + 17°) - \sin(60° - 17°)$

$= 2\cos 60°.\sin 17°$

$= 2\left(\frac{1}{2}\right)\sin 17°$

$= \sin 17°$

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270°;90°]$



a) Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270°;90°]$ on the grid provided. Show ALL the intercepts with the axes, as well as the turning points.

b) Let A be a point of intersection of the graphs f and g. Show that the x-coordinate of A satisfies the equation $\sin x = \frac{-1+\sqrt{5}}{2}$.

c) Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x \in [-270°;90°]$

Teaching notes:

Ask: *What transformation has happened to the standard cosine function?*

(There has been a period change. The new period is 180º).

a)

Ask: *What transformation has happened to the standard sine function?*

(The amplitude has changed to 2 and the function has shifted down 1 unit).

Remind learners that they need to know the standard functions well.

Ask: *What does an amplitude of 2 mean?*

(The height of the function will be 4 units).

Due to the vertical shift, what will the range of the sine function be?

(-3 to 1)

b)

Ask: *How many points of intersection are there?*

(2)

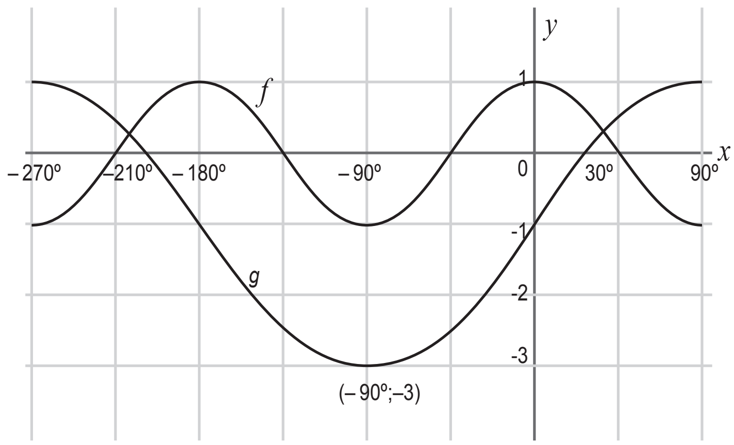
Why does it seem that there is only one in the question?

(Solving a trigonometric equation leads to a general solution then the specific values need to be found).

c)

Tell learners that this is a continuation of the general solution found in the previous question and that they need to use their knowledge of equations to find the two possible solutions. Point out that the solution to this question could help them to check that their drawing of the function looks correct.

a)



b) $\cos 2x = 2\sin x - 1$

$1 - 2\sin^2 x = 2\sin x - 1$

$0 = 2\sin^2 x + 2\sin x - 2$

$0 = \sin^2 x + \sin x - 1$

$\sin x = \dfrac{-b \pm (b^2 - 4ac)}{2a}$

$\sin x = \dfrac{-1 \pm (1^2 - 4(1)(-1)}{2(1)}$

$\sin x = \dfrac{-1 \pm \sqrt{5}}{2}$

$\sin x = 0{,}61803\ldots$ or $\sin x = -1{,}61803\ldots$

N/S

$\therefore \sin x = \dfrac{-1 + \sqrt{5}}{2}$

c) $\sin x = \frac{-1+\sqrt{5}}{2}$

$RA = 38{,}17°$

$\therefore x = 38{,}17° + k.360°$ or $x = 180° - 38{,}17° + k.360°$; $k \in Z$

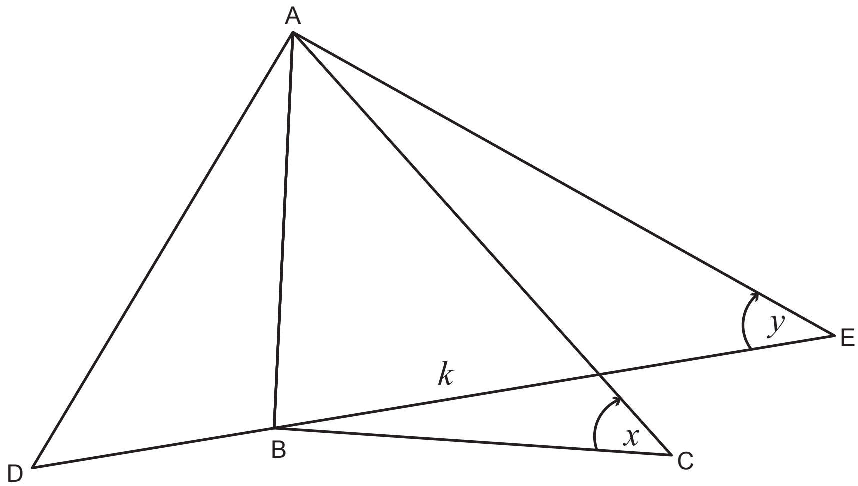
$x = 141{,}83° + k.360°$

$\therefore x = 38{,}17°$ or $-218{,}17°$

$y = 0{,}24$

$\therefore$ Points of intersection: (38,17°;0,24) & (-218,17°;0,24)

AB represents a vertical netball pole. Two players are positioned on either side of the netball at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, d and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



a) Write down the size of $A\hat{B}C$

b) Show that $AC = \frac{k.\tan y}{\sin x}$

c) If it is further given that $D\hat{A}C = 2x$ and $AD = AC$, show that the distance DC between the players at D and C is $2k \tan y$

Teaching notes:

a)

It is important that learners note where there is a right angle when a real-life situation is depicted.

b)

Ask: *Do we have enough information in the triangle with AC in it ($\triangle ABC$)to answer the question?*

(No).

Which triangle can we work in where there is enough information to find a value that can be used in the triangle required?

($\triangle ABE$).

c)

Ask learners to draw the triangle of interest and fill in all the known/given information.

Ask: *What rule can we use to find CD?*

(The sine rule can be used).

Remind learners that they need to find CD and not use the solution given.

---

a) $A\hat{B}C = 90°$

b) In $\triangle ABE$:

$$\tan y = \frac{AB}{BE}$$

$$\tan y = \frac{AB}{k}$$

$$AB = k\tan y$$

In $\triangle ABC$:

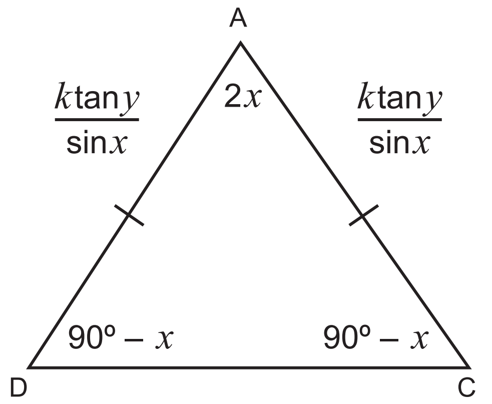
$$\sin x = \frac{AB}{AC}$$

$$\text{AC} \sin x = AB$$

$$\text{AC} = \frac{AB}{\sin x}$$

$$\therefore \text{AC} = \frac{k\tan y}{\sin x}$$

c) In ΔADC:



$\hat{D} = \hat{C} = 90^\circ - x$ (<'s of Δ)

$$\frac{DC}{\sin 2x} = \frac{AC}{\sin(90^\circ - x)}$$

$$\frac{DC}{\sin 2x} = \frac{\frac{k\tan y}{\sin x}}{\cos x}$$

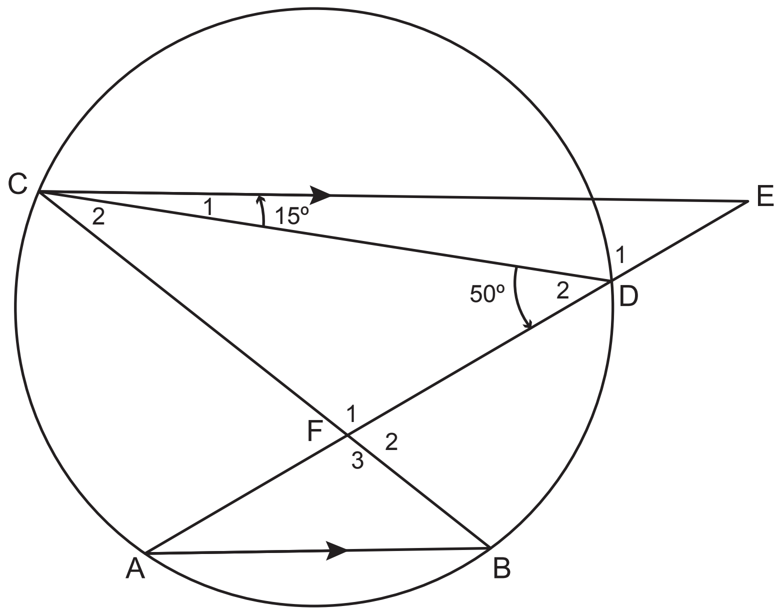
$$DC = \frac{\frac{k\tan y}{\sin x}.2\sin x\cos x}{\cos x}$$

$$DC = \frac{k\tan y.2\cos x}{\cos x}$$

$$DC = 2k\tan y$$

EUCLIDEAN GEOMETRY and MEASUREMENT

In the diagram, points A, B, D and C lie on a circle. $CE \parallel AB$ with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2$ = 50º and $\hat{C}_1$ = 15º.



a) Calculate, with reasons, the size of:

(i) $\hat{A}$

(ii) $\hat{C}_2$

b) Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E.

Teaching notes:

a)

Both (i) and (ii) require knowledge of the basic circle theorems from Grade 11. Learners should be able to answer these easily.

If learners are struggling with these theorems show how the chord is the starting point and show the angles subtended into the same segment that are equal.

b)

Remind learners that to prove a tangent they will either need to use the converse of the tangent-chord theorem or prove that the radius is perpendicular to the tangent.

Ask: *What will we use to prove the tangent?*

(Converse tan-chord).

Ask: *What other theorem will be required?*

(Exterior angle of triangle).

Solutions:

a) (i) $\hat{B} = 50°$ (<'s in same segment)

$\hat{C}_2 = 35°$ ($CE \parallel AB$; alt <'s equal)

$\therefore \hat{A} = 35°$ (<'s in same segment)

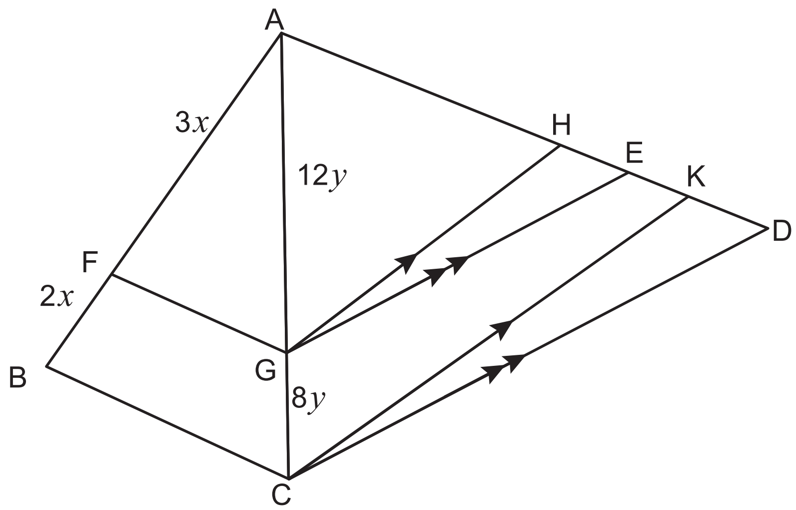
(ii) $\hat{C}_2 = 35°$ (<'s in same segment)

b) $\hat{E} = 35°$ (ext < of Δ)

$\therefore \hat{E} = \hat{C}_2$

$\therefore$ CF is a tangent (converse tan chord therorem)

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that $AF = 3x$, $FB = 2x$, $AG = 12y$ and $GC = 8y$. H, E and K are points on side AD such that $GH \parallel CK$ and $GE \parallel CD$.



a) Prove that:

(i) $FG \parallel BC$

(ii) $\frac{AH}{HK} = \frac{AE}{ED}$

b) If it is further given that $AH = 15$ and $ED = 12$, calculate the length of EK.

Teaching notes:

a) (i) and (ii)

Ask: *What theorem will be required to prove that the lines mentioned are parallel lines and that the sides are in proportion?*

(Proportion theorem – line divides two sides proportionally).

Confirm with learners that they know which triangles to focus on.

b)
Tell learnes to fill the information given onto the diagram. This should enable them to see what triangles may be used.
Ask learners to note that AH is linked to triangles AGH and ACK, while EG is linked to triangles AGE and ACD.
All of these have a direct link to the proportion of AG and CG.
This information, as well as some basic arithmetic regarding which length subtract another length, will give a length required should assist learners in answering the question.
It is not easy, so many learners will require your assistance.

a) (i) $\frac{AF}{BF} = \frac{3x}{2x} = \frac{3}{2}$

$\frac{AG}{CG} = \frac{3y}{2y} = \frac{3}{2}$

$\therefore \frac{AF}{BF} = \frac{AG}{CG}$

$\therefore FG \parallel BC$ (line divides 2 sides of Δ in proportion)

(ii) $\frac{AG}{CG} = \frac{AH}{HK}$ (proportion theorem; $GH \parallel CK$)

$\frac{AG}{CG} = \frac{AE}{DE}$ (proportion theorem; $GE \parallel CD$)

$\therefore \frac{AH}{HK} = \frac{AE}{DE}$

b) $\frac{AH}{HK} = \frac{AE}{DE} = \frac{3}{2}$

$\frac{15}{HK} = \frac{3}{2}$

$3HK = 30$

$HK = 10$

$\frac{AE}{DE} = \frac{3}{2}$

$\frac{AE}{12} = \frac{3}{2}$

$2AE = 36$

$AE = 18$

$AE - HE = AH$

$18 - HE = 15$

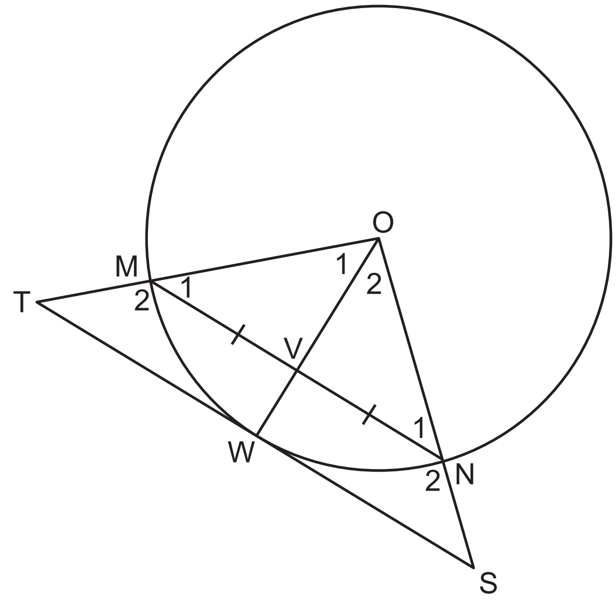
$\therefore HE = 3$

$HK - HE = EK$

$10 - 3 = EK$

$\therefore EK = 7$

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that $MV = VN$. The tangent at W meets OM produced at T and ON produced at S.



a) Give a reason why $OV \perp MN$.

b) Prove that:

(i) $MN \parallel TS$

(ii) TMNS is a cyclic quadrilateral

(iii) $OS.MN = 2ON.WS$

Teaching notes:

a)

Remind learners they need to know their theory well.

b)

(i) Ask: *How can we prove any two lines parallel?*

(Corresponding angles equal; alternate angles equal or co-interior angles supplementary).

Ask: *Which of these options can be used?*

(corresponding or co-interior angles)

(ii) Ask: *How can we prove that a quadrilateral is cyclic?*

(Opposite angles supplementary; exterior angle equal to opposite interior angle; equal angles from same line).

Ask: *Which of these options can be used?*

(Opposite angles supplementary or exterior angle).

(iii) Advise learners to consider the multiplication given and reverse it to find the sides that should be in proportion.

This should assist learners in finding which triangles to prove similar in order to prove the proportion and hence the products required.

Many learners will need assistance with this question. Using the fact that $MV = VN$ and changing it into $VN = \frac{1}{2}MN$ may not be easy for everyone to see.

Solutions:

a) Line from centre to midpoint chord

b) (i) $OV \perp MN$ (line from centre to midpoint chord)

$OW \perp TS$ (radius $\perp$ tangent)

$\therefore MN \parallel TS$ (corres <'s = 90º)

(ii) $\hat{M}_1 = \hat{T}$ (corres <'s equal; $MN \parallel TS$)

$\hat{M}_1 = \hat{N}_1$ (equal <'s oppos equal sides; radii)

$\therefore \hat{T} = \hat{N}_1$

$\therefore TMNS$ is a cyclic quad (ext < = opp int <)

(iii) In ΔOVN and ΔOWS

$\hat{O}_2 = \hat{O}_2$ (common)

$O\hat{V}N = O\hat{W}S = 90º$ (proved above)

$\hat{N}_1 = \hat{S}$ (sum <'s Δ)

$\therefore \Delta OVN \,|||\, \Delta OWS$ (AAA)

$$\therefore \frac{VN}{WS} = \frac{ON}{OS}$$

But $VN = \frac{1}{2}MN$

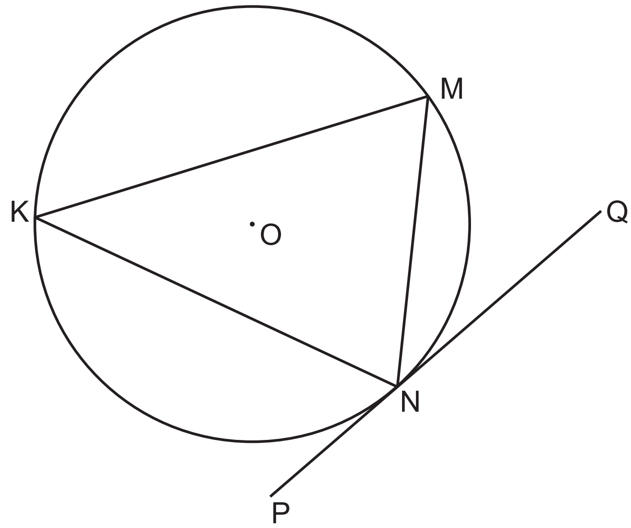
$$\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$$

$\therefore WS.ON = \frac{1}{2}MN.OS$

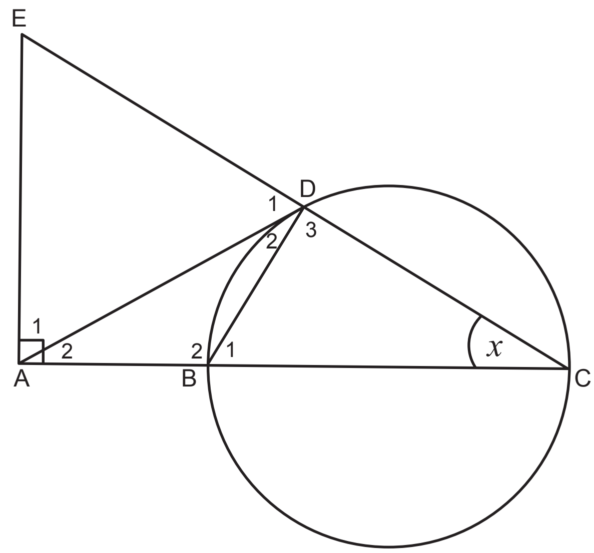
$\therefore 2WS.ON = MN.OS$

a) In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.

Prove the theorem which states that $M\hat{N}Q = \hat{K}$



b) In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that $EA \perp AC$. BD is drawn.
Let $\hat{C} = x$.



(i) Give a reason why:
  a) $\hat{D}_3 = 90°$
  b) ABDE is a cyclic quadrilateral
  c) $\hat{D}_2 = x$

(ii) Prove that:
  a) $AD = AE$
  b) $\Delta ADB \mathbin{|||} \Delta ACD$

(iii) It is further given that $BC = 2AB = 2r$.
  a) Prove that $AD^2 = 3r^2$
  b) Hence, prove that ΔADE is equilateral.

Teaching notes:

a)

Discuss again with learners the importance of knowing their theory and that if theory is asked, this theorem is going to be required to answer any further questions.

b)

(i) All three questions should be straightforward for learners – basic theory.

(ii) a) Ask: *How can we prove that two sides of a triangle are equal?*

(By proving two angles equal or proving two different triangles congruent).

Which method can be used to answer this question?

(Proving two equal angles).

b) Ask: *How can we prove two triangles similar?*

(Three equal angles or sides in proportion).

Which one can be used to answer this question?

(Three equal angles).

(iii) Point out that ‘it is further given’ is suggesting that anything previously proved will be useful to answer the question that follows.

Tell learners to fill in the information given ($BC = 2r$ and $AB = r$).

a) Ask: *If $\triangle ADB /// \triangle ACD$, which sides will be in proportion? Write these down.*

Ask: *Will this be useful to answer the question?*

b) Ask: *How can we prove that a triangle is equilateral?*

(All three angles must equal 60º).

Ask: *What do you already know about $\triangle ADE$?*

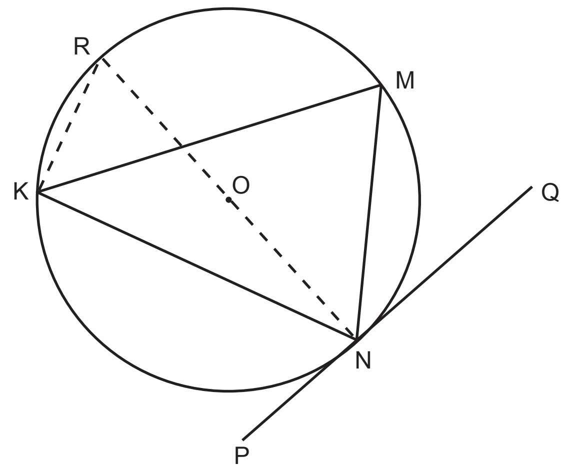
($AD = AE$).

Tell learners that all we need to do is prove.

There are several ways of answering this question. Using trigonometry may be the most obvious. Time permitting, discuss more than one method with learners.

Solutions:

a) Construction: Draw in diameter NR and join R to K.



RTP: $M\hat{N}Q = \hat{K}$

PROOF:

$R\hat{K}M + M\hat{K}N = 90°$	(< in semi-circle)
$R\hat{N}M + M\hat{N}Q = 90°$	(rad $\perp$ tan)
$R\hat{K}M = R\hat{N}M$	(<'s in same segment)
$\therefore M\hat{K}N = M\hat{N}Q$	
$\therefore M\hat{N}Q = \hat{K}$	

b)

(i) a) Angle in semi-circle

b) Exterior angle = opp interior angle

c) Tan chord theorem

(ii) a) In ΔACE, $\hat{E} = 90° - x$ (sum <'s Δ)

$\hat{D}_2 = x$	(tan chord theorem)
$\therefore D_1 = 90° - x$	(<'s on str line)
$\therefore \hat{E} = \hat{D}_1$	
$\therefore$ AD = AE	(equal sides opp equal angles)

b) In ΔADB and ΔACD

$\hat{A}_2 = \hat{A}_2$	(common)
$\hat{D}_2 = \hat{C}$	(proved above)
$\therefore A\hat{B}D = A\hat{D}C$	(sum <'s of Δ)
$\therefore \Delta ADB \;///\; \Delta ACD$	(AAA)

(iii) a) $\dfrac{AD}{AC} = \dfrac{AB}{AD}$ ($\Delta ADB \;///\; \Delta ACD$)

$\therefore AD^2 = AC.AB$

$= 3r.r = 3r^2$

b)	$AD = AE$	(proved above)
	$AD^2 = 3r^2$	
	$\therefore AD = AE = \sqrt{3}r$	
	$AC = 3r$	(given)
	In ΔACE:	
	$\tan \hat{E} = \frac{AC}{AE}$	
	$= \frac{3r}{\sqrt{3}r}$	
	$= \frac{3}{3}$	
	$\therefore \hat{E} = 60^\circ$	
	$\therefore \hat{D} = 60^\circ$	($AD = AE$)
	$\therefore \hat{E} = 60^\circ$	(sum <'s of Δ)
	$\therefore \Delta ADE$ is equilateral	

8. When the past paper has been completed ask learners if they have any questions.

9. Say: *Next week you will work through past papers on your own.*

TERM 4

REVISION - WEEK 3

A POLICY AND OUTCOMES

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Lesson Objectives

By the end of the lesson, learners will have:

- completed a Paper 1 past paper
- completed a Paper 2 past paper
- made ‘cheat sheets’ covering all topics.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the past examination papers that the learners will be doing on their own. This is essential to assist them quickly and smoothly when learners need help.
   - work through the instructions to learners on how to make a cheat sheet to assist them in their studying.
3. The examinations and memoranda are available in the Resource Pack (Resources 5 -8) for photocopying if possible.
4. Write work on the chalkboard before the learners arrive to ensure no time is wasted.

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Learners have now done revision with your help. It is time for them to try past papers on their own.
2. As learners work through the past papers they should make ‘cheat sheets’. Explain how to go about this before they start on the past papers.

DIRECT INSTRUCTION

1. Start the lesson by saying: *Now that we have spent two weeks doing revision together, it is time for you to work on your own. I will be available to assist you but mostly you need to work alone.*
2. Tell learners that while they are working through the papers, they should make cheat sheets. Make it clear that cheat sheets are a study aid and not notes that can be used for cheating!
3. Go through the following instructions with learners.

> A cheat sheet is a document (generally only one-page-front and back) that contains all the key information that is likely to be in an assessment. Even though you can't use the cheat sheet in the examination, the preparation of a cheat sheet is a great way to prepare for the exam.
>
> Guidelines for preparing a cheat sheet:
>
> 1. Develop the cheat sheet gradually by adding new items as you work through past papers.
> 2. Write out the cheat sheet by hand. You can get more on the document that way. At the end of each past paper, re-do the cheat sheets for each topic and put them in a safe place with your summary notes as well as any other study notes.
> 3. Include the following items on your cheat sheet:
>   - Formulas
>   - Example problems worked out
>   - Steps used in the problem listed in order
>   - Reminders of things to look out for in doing a problem
>   - Rules used to solve problems
>   - Definitions
>   - Types of problems that you know will be in an examination.
> 4. If you recall problems you struggled with in the past, be sure to include information on these.
> 5. Use your past papers as a guideline to prepare your cheat sheets. Past papers are all set along the same lines. This is your 2nd past paper for each exam (Paper 1 and Paper 2), so you should start noticing what is often assessed.
> 6. While you are working through the past paper, refer to your exercise book, text book and summary notes for cheat sheet information.
> 7. Find a method to compartmentalise items. For example, highlight what you need to memorise in one colour and tips in another colour. Use bullet points and different-sized headings. Find a layout that suits your study method.
> 8. Choose whether your cheat sheet is a summarised list (like the summary notes you have already received) or a mind map.

9. Review your cheat sheet and summary notes for at least one hour every day for a week before the exam. This continual review will help you remember the concepts.
10. Use your cheat sheet as the primary study resource for the final. If you have kept these up-to-date, you should be able to reduce your preparation time for finals.

4. Hand out both the past papers' exemplars (Paper 1 and Paper 2 – Resources 5 and 6). Allow learners to choose where they start. Point out, however, that by the end of the 7 or 8 days they must have completed both papers as well as their cheat sheets. You will need a few days for the past papers to be marked and corrected.

5. If photocopying is available, photocopy a few memoranda (Resources 7 and 8) and, with 2 or 3 days to go, allow learners to sit in groups to mark and discuss the solutions. If photocopying is not an option, learners could be given the link (providing some of them have data) and marking could still take place in groups.

6. If neither of these is possible, call out the final answers of each question then ask learners which they would like you to do in full with them.

7. Verbalising problems and sharing their own knowledge with others can be a very effective learning tool.

8. The correcting of the papers is an essential part of the exercise. There is little value in doing questions and not knowing if they have been done correctly or not. When marking does take place, tell learners that they must do the full corrections for any question that was incorrect.

9. Learners should add notes to their cheat sheets relating to any mistakes they made.

10. We wish you and your learners well.